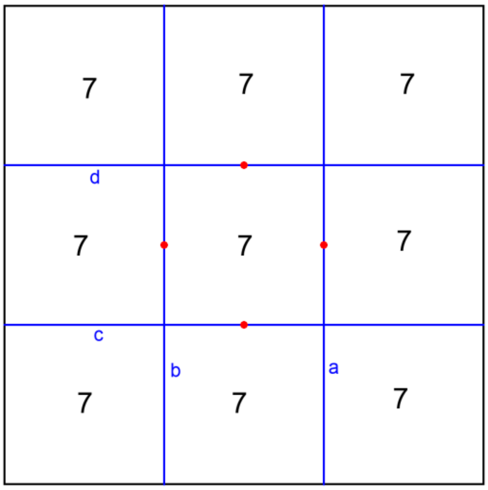
Third-Order Linear Area Magic Square – algorithm

Walter Trump, 2017-01-10

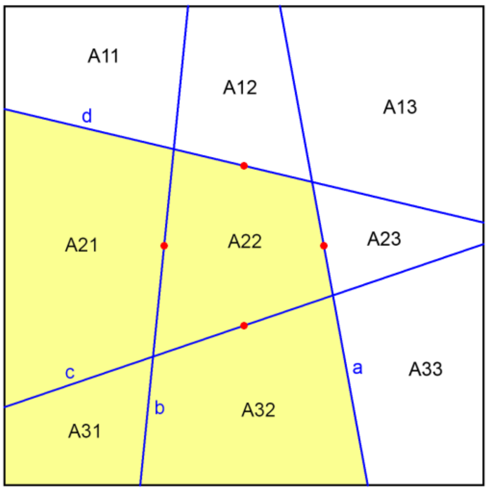
Area magic squares are an idea of William Walkington - December 2016.

Example with numbers from 3 to 11 and magic sum $3 \cdot 7 = 21$

6	5	10
11	7	3
4	9	8



We start with 9 equal areas of size 7.
The four red fixpoints are the centers of the four lines a, b, c and d.
The sides of the small squares have the length $\sqrt{7}$.



The sizes of the areas can be varied by rotating the lines a, b, c and d around the red fixpoints.

a and b split the square in 3 trapezoids (the columns of the magic square). c and d split the square in 3 other trapezoids (the rows of the magic square).

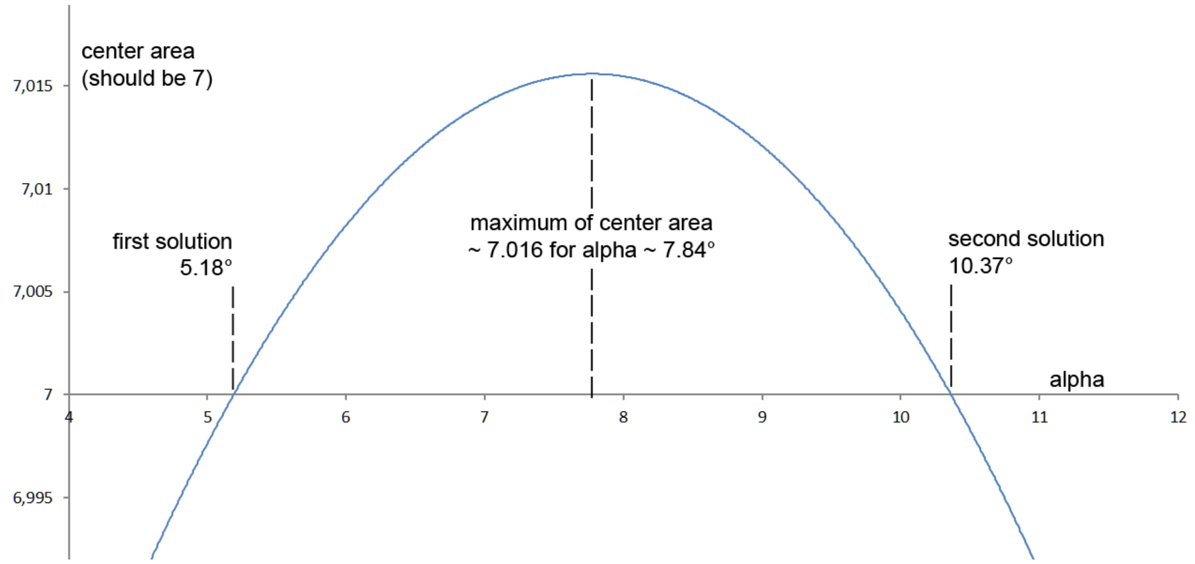
The sizes of the areas of the 6 trapezoids do not change and the areas always make a semi-magic square.
(For example: $A11 + A21 + A31 = (3 \cdot \sqrt{7}) \cdot \sqrt{7} = 21$)

We only consider the 4 yellow areas and try to achieve the correct values for the areas.
In a valid solution the other 5 areas are automatically correct.
(See appendix for the definitions of the angles α, β, γ and δ.)

We vary angle α, which describes the rotation of line a from its vertical starting position.
The size of (A31 + A32) depends only on the lines a and c. (A31 + A32) should be equal to $4 + 9 = 13$.
For the actual value of α we determine the angle γ (line c) for which the size of (A31 + A32) is equal to 13.

As we know c now we can in the next step determine β in order to achieve $A31 = 4$.
In step 3 we calculate δ by using $(A21 + A31) = 11 + 4 = 15$ and the known orientation of b.

With the actual values of α and δ we can calculate the size of area A21+A22+A31+A32.
By subtracting (11 + 4 + 9) from the actual value of A21+A22+A31+A32 we get the actual value of A22.
Of course we cannot expect that A22 (the center area) has already the desired size 7.
Therefore we have to do the procedure several times. In this way we can study A22 as function of α.



For some values of α the size of A22 is greater than 7. The maximum size is about 7.016 for $\alpha \approx 7.84°$.
We find the correct values for α by decreasing α until A22 = 7 or by increasing α until A22 = 7.
In this way we get two solutions in general. (*In fact a more effective iteration is used in the program*.)

A solution is not valid if the lines a and b or the lines c and d intersect within the square.

Appendix: Definition of the angles